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Analytical investigation of the armature current influence on the torque and radial force in eccentric consequent-pole PM machines

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Abstract
In this work, the electromagnetic torque and the unbalanced magnetic force (UMF) in eccentric consequent pole PM (CPPM) machines are investigated. For this investigation, in the first part of the paper, an exact analytical model is provided for the CPPM machine by solving the Poisson equation in the eccentric and salient air gap. In the analytical modelling process, the concept of equivalent magnetizing current (EMC), the subdomain technique, and a bilinear transformation are applied. The obtained model is verified by means of FEA. In the second part of the paper, by using the developed model, the relationship between the machine electromagnetic torque and the UMF to the d- and q-axis components of the armature current is studied. It is shown that in CPPM machines, the influence of the q-axis current on the electromagnetic torque is much higher than the effect of the d-axis current. On the other hand, the eccentric machine UMF is strongly dependent on the d-axis armature current while the variation of the UMF due to the changes in the q-axis current is small. Therefore, in eccentric CPPM machines, UMF could be suppressed by injecting a negative d-axis armature current (flux weakening).

1 INTRODUCTION

PM machines have attracted great interest because of the emerging high-power density permanent magnet materials and progress in power electronics devices and processors. Different types of rotor geometries exist in the PM machines including surface-mounted PM (SPM), PM-inset, consequent pole PM (CPPM), interior PM, and buried PM machines. In addition, in other types of PM machines such as flux-switching, and flux reversal machines, the PMs are located inside the stationary part of the machine. Traditionally, because of the simple structure of SPM, PM-inset, and CPPM machines, they are more preferred to be used over the other types of PM machines. The structure of a radial flux CPPM machines is shown in Figure 1a. As seen in Figure 1a, the geometry of CPPM machine is almost the same as that of the PM-inset machines, while the PM arrangement includes similar poles for the PMs. For the CPPM machines the axial flux [1] and transversal flux [2] structures are also proposed. There are some benefits for the CPPM machines over the surface-mounted PM machines including:

- Lower volume of the PM usage and lower overall cost of CPPM [3].
- Lower eddy currents in the PM blocks [4], and higher flux weakening capability [5] as well as the reluctance torque [6] due to interaction of the rotor tooth with armature field.
- Lower fringing leakage flux which results in higher flux linkage and consequently higher torque and power density [7].

However, the CPPM machines include some drawbacks such as high torque ripple [8], unipolar leakage flux and, consequently, unbalanced magnetic path [9]. To investigate the characteristics of CPPM, FEA as a powerful and precise tool is applied to analyse the CPPM machine [3,4,8–10]. The FEA approach is able to consider geometrical details as well as the material non-linearity. However, FEA is a time-consuming procedure and requires a high computational burden, which is
not appropriate for the first design steps of electrical machines. In contrast, the analytical models are fast and easy to use. Against, several 2D exact analytical models that are proposed for SPM and PM-inset machines [11,12], few numbers of researches provide exact analytical models for CPPM machines. Based on the subdomain technique, exact 2D models are provided for the inner [13] and outer rotor [14] CPPM machines. However, these models do not include the rotor eccentricity which is a common fault in electric machines. The rotor eccentricity of the SPM machines is modelled by means of perturbation analysis [15,16], equivalent magnetizing currents [17], and conformal transformation [18] methods. An exact analytical model of the eccentric and salient air gap of PM-inset machines is provided in ref. [19]. Against, variety of analytical models proposed for considering of the rotor the eccentricity in different types of PM machines, no analytical model addresses the eccentric CPPM machines, based on the authors' knowledge. However, the rotor eccentricity of the CPPM machines is investigated by means of FEA in ref. [20]. This gap is filled in Section 2 of this paper by providing an exact 2D analytical model for the eccentric CPPM machines based on the Poisson solution. In the modelling process, the concept of the equivalent magnetizing current (EMC), subdomain technique, and a bilinear transformation are applied in Section 2. In Section 3, the obtained model is verified by means of FEA.

Due to the large air gap seen by the armature in the eccentric SPM machines, the influence of the armature current on the produced UMF is too small. In other words, significant part of the generated UMF is due to the magnetic pull between the PMs and the stator ferromagnetic body. In the other hand, due to the small and salient air gap in the eccentric CPPM machines, the armature could have a considerable impact on the UMF. Due to this characteristic of CPPM machine, special stator winding is proposed for the bearingless CPPM machines [21,22]. The machines in refs. [21,22] have no bearings but they include two sets of winding for controlling the torque and UMF separately. The torque and UMF windings have different pole numbers. In contrast, in this paper, it is supposed that the CPPM machine includes one set of P-pole stator winding as well as bearings. In other words, the electromagnetic torque and the

![Image of eccentric CPPM machine](image_url)
The effect of armature current on the resultant magnetic field at the eccentric stator bore by the PMs and EMC must be replaced with some EMC at its surface [24]. If EMC is selected appropriately, the boundary condition at the stator bore is satisfied. In other words, the produced tangential field at the eccentric stator bore by the PMs and EMC must be \(-\mu_0 J_{ar}\), where, \(J_{ar}\) is the current density of the armature winding considered in the slot openings. Actually, the distribution of the EMC is considered as Fourier expression with unknown coefficients and is found by applying the boundary conditions. Hereafter, the above described procedure is followed to obtain the air gap flux density in the eccentric CPPM machines.

2.2 Subdomain analysis and its application to eccentric machines

As a well-known method, the separation of variables technique is used as an analytical approach to find the solution of Poisson's equation in magneto-static problems. The coefficients of the solution must be found such that the considered boundary conditions are satisfied. However, this approach could be used in the problems with simple geometries in the Cartesian or cylindrical coordination systems, for example, the rectangular or ring geometries. In case of having more complex geometries such as slotted ring-shape geometries, other techniques such as using conformal transformation, subdomain method or EMC must be used to find the solution. However, the conformal transformation could be applied only on the Laplace equations while this limitation is not a matter in subdomain analysis. In the subdomain analysis method, the complex geometry is divided into some sub-areas which have simple geometries. In other words, the boundaries of the considered sub-areas have simple expressions in the considered coordinate, for example, constant radius or constant angle in the cylindrical coordination system. Therefore, the Poisson's solution based on the separation of variables method could be expressed in each individual sub-area very easy. However, each sub-area has its own coefficients. Considering the boundary conditions of all sub-areas, linear functions of the used coefficients are obtained. Therefore, the used coefficients are obtained easily by solving the obtained linear equations.

In the case of having an eccentric and sotted air gap, the Poisson equation could not be solved directly by using subdomain analysis. The reason is that the eccentric air gap could not be decamped to sub-area with simple geometries. On the other hand, although conformal transformations could convert the eccentric air gap to a concentric geometry, this method is not allowed to be used in the problems with the Poisson equation as the governing equation. This problem is solved by using the EMC method.

2.3 PM flux density

To compute the flux density of the PMs in Figure 1b the subdomain technique is used. The defined regions and the governing equations in the regions are given in Equations (1) and (2), respectively.
where, \(R_s, R_r, \) and \(R_m\) are radii of the stator, rotor, and magnet, respectively, \(M\) is the PM magnetization vector, \(N_p\) is the number of rotor slots, and \(\varphi_{k}\) and \(\varphi_{k}^{s}\) are the angular limits of the \(k\)th rotor slot area as shown in Figure 1b. Since there is even-symmetry in the radial flux density in the air gap region (region 1), the general solution of the magnetic potential vector in this region is obtained as Equation (3).

\[
A_{I} = \sum_{p} d_{p} r^{-\varphi} \sin np\varphi
\]

where, \(p\) is the number of the machine pole pairs. The boundary conditions are illustrated in Figure 1c. Considering the first rotor slot region (region 2), there is zero tangential flux density at the circumferential borders \((B_{r} = 0 \text{ for } \varphi = \pm \alpha, \pi/\alpha)\) as well as the odd symmetry in the circumferential flux density distribution. Therefore, the general solution of the magnetic potential vector in this region is expressed as given in Equation (4).

\[
A_{II} = \sum_{n} (b_{n} r^{\varphi} + c_{n} r^{-\varphi} + K_{n}\varphi) \sin np\varphi
\]

where, \(K_{n}\) is considered as the particular solution. From mathematical point of view, the particular solution must include all the possible solutions for the non-homogeneous partial differential equation in Equation (2). The parameter \(K_{n}\) is given in Equation (5), where \(B_{\text{rem}}\) is the PM remanence and \(\alpha_{r}\) and \(\alpha_{p}\) are the ratio of the rotor slot and PM arcs to the pole pitch, respectively.

\[
K_{n} = \frac{4B_{\text{rem}}\alpha_{r}^{2}}{\mu_{0}n\pi(\alpha_{p}^{2} - (n\pi)^{2})} \sin \left(\frac{n\pi \alpha_{p}}{2 \alpha_{r}}\right)
\]

Applying zero circumferential field at \(r = R_r\) in the rotor slot regions, the parameter \(b_{n}\) is obtained as Equation (6).

\[
b_{n} = c_{n} R_{r}^{\alpha_{r}} - K_{n} R_{r}^{\alpha_{r} + 1}
\]

The unknown coefficients \(c_{n}\) and \(a_{n}\) are obtained by considering the continuity conditions at \(r = R_m\) as given in Equation (7).

\[
\frac{\partial A_{I}}{\partial r} |_{r=R_m} = \frac{\partial A_{II}}{\partial r} |_{r=R_m}
\]

\[
\frac{\partial A_{I}}{\partial \varphi} |_{r=R_m} = \frac{\partial A_{II}}{\partial \varphi} |_{r=R_m}
\]

Applying the correlation technique [11] on the equalities in Equation (7), the coefficients \(a_{n}\) and \(c_{n}\) are obtained as Equation (8).

\[
\begin{bmatrix}
c \\
a \end{bmatrix} = \begin{bmatrix} D & E \end{bmatrix}^{-1} \begin{bmatrix} H \\
F & G \end{bmatrix}
\]

where \(c\) and \(a\) include \(N_{\text{max}}\) and \(V_{\text{max}}\) number of the harmonics of \(c_{n}\) and \(a_{n}\), respectively, \(D\) and \(G\) are diagonal matrices. The element of Equation (8) are reported in Equation (9), where the functions \(S(n,v)\) and \(T(n,v)\) are given in Equation (10).

\[
D_{n,v} = \frac{R_{m}^{\alpha_{v}} R_{r}^{\alpha_{v}}}{R_{r}^{\alpha_{v} - \alpha_{p}} + n\pi R_{m}^{\alpha_{v} - \alpha_{p}}}
\]

\[
E_{n,v} = -v p R_{m}^{\alpha_{v}} S(n,v)
\]

\[
H_{n,v} = \frac{1}{2} K_{n} \left(\frac{R_{m}^{\alpha_{v}}}{R_{r}^{\alpha_{v}}} - R_{r}^{\alpha_{v}} \right)
\]

\[
L_{n,v} = \sum_{n=1}^{N_{\text{max}}} \frac{n p K_{n}}{\alpha_{r}} \left(\frac{R_{m}^{\alpha_{v} - 1}}{R_{r}^{\alpha_{v}}} + 1\right) T(n,v)
\]

\[
F_{n,v} = \frac{n p}{\alpha_{r}} \left(\frac{R_{m}^{\alpha_{v} - 1}}{R_{r}^{\alpha_{v}}} R_{r}^{\alpha_{v} - \alpha_{p}} - R_{m}^{\alpha_{v} - \alpha_{p}}\right) T(n,v)
\]

\[
G_{n,v} = \frac{1}{2} v p R_{m}^{\alpha_{v} - 1}
\]

\[
S(n,v) = \int_{-\alpha_{r}}^{\alpha_{r}} \cos(np\varphi) \cos \left(\frac{np\varphi}{\alpha_{r}}\right) d\varphi
\]

\[
T(n,v) = \int_{-\alpha_{r}}^{\alpha_{r}} \sin(np\varphi) \sin \left(\frac{np\varphi}{\alpha_{r}}\right) d\varphi
\]

Finally, the radial and circumferential air gap flux density components due to the PMs yield as Equation (11).

\[
B_{r,\text{PM}} = \sum_{n=1}^{N_{\text{max}}} - a_{n} v p R_{m}^{\alpha_{v} - \alpha_{p}} \cos np\varphi
\]

\[
B_{\varphi,\text{PM}} = \sum_{n=1}^{N_{\text{max}}} - a_{n} v p R_{m}^{\alpha_{v} - \alpha_{p} - 1} \sin np\varphi
\]

### 2.4 EMC field distribution

The transformation in Equation (12) is used to make the eccentric geometry in the \(Z\)-plane (Figure 1a) to a concentric one in the \(W\)-plane (see Figure 2), where, \(z\) and \(w\) are the complex variables in the \(Z\) - and \(W\)-plane, respectively, and \((x,y)\) and \((u,v)\) are the Cartesian components in the \(Z\)- and \(W\)-plane, respectively, \(D\) is the off-centre distance as shown in Figure 1a.
The considered regions in the W-plane and the boundary conditions at the radius $\rho = \rho_s$ are given in Equations (14) and (15), respectively. The boundary conditions are illustrated in Figure 2b.

Region I : $\rho_m < \rho < \rho_s$
Region $k, k \in \{2, 3, \ldots N_r + 1\}$ : $\theta_{ke} < \theta < \theta_{ke'}$ and $\rho_r < \rho < \rho_m$
Region $N_r + 2$ : $\rho_s < \rho$

\[
\frac{\partial A_l^W}{\partial \theta} = 0
\]

\[
\frac{\partial A_1^W}{\partial \theta} = \mu_0 J_E^W
\]

The governing equation on all considered regions is Laplace equation. The general solution of the magnetic field vector in the regions I and $N_r+2$ are given in Equation (16).

\[
A_l^W = \sum \left( \frac{\mu_0 J_{Em}^*}{2m} \rho \left( \rho \rho_s \right)^m \right) \cos \theta
\]

\[
+ \sum \left( \frac{\mu_0 J_{Em}^*}{2m} \rho \left( \rho \rho_s \right)^m \right) \sin \theta
\]

The boundary conditions in the rotor slot regions in the W-plane are given in Equation (17).

\[
\left. \frac{\partial A_k^W}{\partial \rho} \right|_{\rho=\rho_s} = 0
\]

\[
\left. \frac{\partial A_k^W}{\partial \theta} \right|_{\theta=\theta_0} = \left. \frac{\partial A_k^W}{\partial \theta} \right|_{\theta=\theta_w} = 0
\]

Considering the boundary conditions (Equation 17), the general solution of the magnetic field vector in the rotor slot regions are given in Equation (18).

\[
A_k^W = \sum b_{kn} \left( \rho \rho_s \rho - T_{kn} \right) \cos \frac{\theta - \theta_k}{2}
\]

\[
\gamma_{kn} = \frac{n \pi}{\theta_{ke} - \theta_{ke'}}
\]

Finally, the air gap flux density distribution in the region I are obtained as Equations (19) and (20), respectively.
\[ B_{te}^{*} = \sum_{\nu} \left( \frac{\mu_0 E_{oa}}{2} \left( \frac{\rho}{\rho_s} \right)_{\nu}^{n-1} + a_{4\nu} \rho^{-\nu-1} \right) \sin \nu \theta \]
\[ -\sum_{\nu} \left( \frac{\mu_0 E_{oa}}{2} \left( \frac{\rho}{\rho_s} \right)_{\nu}^{n-1} + a_{4\nu} \rho^{-\nu-1} \right) \cos \nu \theta \]
\[ B_{tE}^{W} = \sum_{\nu} 2 \left( \frac{\mu_0 E_{oa}}{2} \left( \frac{\rho}{\rho_s} \right)_{\nu}^{n-1} - a_{2\nu} \rho^{-\nu-1} \right) \cos \nu \theta \]
\[ + \sum_{\nu} 2 \left( \frac{\mu_0 E_{oa}}{2} \left( \frac{\rho}{\rho_s} \right)_{\nu}^{n-1} - a_{4\nu} \rho^{-\nu-1} \right) \sin \nu \theta \] (19)

Considering the continuity boundary conditions between the rotor slots and the air gap (i.e., \( \rho = \rho_m \)) and using the correlation technique \([2]\), the coefficients \( a_{2\nu}, a_{4\nu} \) and \( b_{\nu} \) are obtained by Equation (21).

\[
\begin{bmatrix}
    a_2 \\
    a_4 \\
    b
\end{bmatrix} = \begin{bmatrix}
    I_1 & I_2 & I_3 \\
    I_4 & 0 & I_5 \\
    0 & I_4 & I_6
\end{bmatrix}^{-1} \begin{bmatrix}
    L_1 & L_2 \\
    L_3 & 0 \\
    0 & L_3
\end{bmatrix} \begin{bmatrix}
    J_{Ec}^{W} \\
    J_{Es}^{W}
\end{bmatrix}
\] (21)

where, \( a_2, a_4, J_{Ec}^{W} \) and \( J_{Es}^{W} \) include \( N_{max} \) number, and \( b \) includes \( V_{max} \) number of the related harmonics, \( I_1, I_4 \) and \( L_3 \) are diagonal matrices with the diagonal elements in Equation (22), and the other matrix indices are given in Equation (23).

\[ I_{3i,j} = \frac{\gamma_{m} T_{m}}{2} \left( 1 + \left( \frac{\rho_r}{\rho_m} \right)^{2 \nu_{m}} \right) \]
\[ I_{4i,j} = -\frac{1}{2} \rho^{-\nu-1} \]
\[ L_{3i,j} = -\frac{\mu_0}{4} \left( \rho_m \rho_s \right)^{n-1} \]
\[ i = 1, 2, ..., (N_r N_{max}) \]
\[ j = 1, 2, ..., V_{max} \]
\[ n = \text{mod}(i, N_{max}) + 1 \] (22)

The other sub-matrix of Equation (21) is defined in Equation (24).

\[ I_{1i,j} = -n \rho_{m}^{-n} T_1(n, \nu, k) \]
\[ I_{2i,j} = -n \rho_{m}^{-n} T_2(n, \nu, k) \]
\[ L_{2i,j} = \frac{\mu_0 \rho_s}{2} \left( \frac{\rho_m}{\rho_s} \right)^{n} T_2(n, \nu, k) \]
\[ L_{2i,j} = \frac{\mu_0 \rho_s}{2} \left( \frac{\rho_m}{\rho_s} \right)^{n} T_2(n, \nu, k) \]
\[ I_{5i,j} = \left( \frac{\rho_r}{\rho_m} \right)^{n} \rho_{m}^{\nu_{m}-1} - \rho_{m}^{\nu_{m}-1} \right) \gamma_{k} T_3(n, \nu, k) \]
\[ I_{6i,j} = \left( \frac{\rho_r}{\rho_m} \right)^{n} \rho_{m}^{\nu_{m}-1} - \rho_{m}^{\nu_{m}-1} \right) \gamma_{k} T_4(n, \nu, k) \] (23)

The used function \( T_1, T_2, T_3 \) and \( T_4 \) are given in Equation (25).

\[
T_1(n, \nu, k) = f_{\theta_k}^o \sin(\nu \theta) \sin \gamma_{k} \theta (\theta_k - \theta_k) d \phi \\
T_2(n, \nu, k) = f_{\theta_k}^o \cos(\nu \theta) \sin \gamma_{k} \theta (\theta_k - \theta_k) d \phi \\
T_3(n, \nu, k) = f_{\theta_k}^o \cos(\nu \theta) \cos \gamma_{k} \theta (\theta_k - \theta_k) d \phi \\
T_4(n, \nu, k) = f_{\theta_k}^o \sin(\nu \theta) \cos \gamma_{k} \theta (\theta_k - \theta_k) d \phi 
\] (25)

2.5 Distribution of EMC

As mentioned before, the distribution of the EMC must be selected such that the boundary conditions at the stator bore, that is, \( J_{ar} \), is satisfied, where the subscripts \( t \) stands for tangential components of the field density and \( J_{ar} \) illustrates the armature current density which is considered in the slot opening areas at the stator bore.

\[ B_{pM} + B_{E} = \mu_0 J_{ar} \] (26)

The tangential field due to the PMs at the stator bore is obtained by Equation (27)

\[ B_r = \sin(\phi_r - \phi_r) B_{PM} + \cos(\phi_r - \phi_r) B_{PVM} \]
\[ \phi_r = \arccos \left( \frac{e}{R_s} - \cos \phi_r \right) \] (27)

Since Equation (26) must be applied in the \( W \)-plane, it is just enough to map the variables \( B_{PM} \) and \( J_{ar} \) from the \( Z \)-to the \( W \)-plane, without trying to obtain the tangential field component of \( J_{E} \) in the \( Z \)-plane. To map the functions \( J_{ar} \) and \( B_{PM} \) from the \( Z \)-to the \( W \)-plane, they are obtained at some discrete points and their values at each point are mapped by using Equation (28), where, \( N_r \) is the number of considered discrete points and the subscript \( i \) is responsible for the \( i \)th segment.

\[ J_{i-ar} = J_{i-ar} \left( \frac{dZ}{dW} \right)^* \]
\[ B_{i-ar}^{W} = B_{i-ar}^{W} \left( \frac{dZ}{dW} \right)^* \] (28)

The \( B_{PM}^{W} \) and \( J_{ar}^{W} \) in the \( W \)-plane are in the discrete form and must be expressed in the form of Fourier series. The distribution of the tangential field due to EMC is presented \( (20) \) versus the unknown Fourier coefficients of EMC. Using Equation (20) and the desired tangential field for EMC in Equation (26), the Fourier coefficients of EMC are obtained as expressed in Equation (29).
where, $Y$ and $Z$ are diagonal matrices with elements given in Equation (30), $B_{\theta PMe}$, $B_{\phi PMe}$, $J_{arc}$, and $J_{arc}$ include the sub-matrices that link the coefficients $Y_{s}$ and $Z_{s}$ to the coefficients $a_{2}$ and $a_{4}$ in Equation (21).

\[
Y_{s} = \frac{\mu_{0}}{2} \left( \frac{\rho_{m}}{\rho_{r}} \right)^{s-1} \]

\[
Z_{s} = \nu \rho_{m}^{s-1} \Theta \]

where, $a_{s}$ and $a_{q}$ are the unitary radial and circumferential vectors in the $Z$-plane, respectively.

### 2.6 Flux density in the Z-plane

Since the Fourier components of EMC are recognized in the W-plane as Equation (29), the boundary condition in Equation (26), is satisfied.

### 2.7 Torque and UMF

Using the Maxwell stress definition, the electromagnetic torque, and the x- and y-components of UMF are obtained as reported in Equations (32) and (33), respectively.

\[
T_{e} = \frac{\gamma_{L_{0}}}{\mu_{0}} \int_{0}^{\pi} B_{s} B_{q} \, d\phi
\]

\[
F_{x} = \frac{\gamma_{L_{0}}}{2\mu_{0}} \int_{0}^{\pi} \left( B_{s}^{2} - B_{q}^{2} \right) \cos(\phi) - 2B_{s}B_{q} \sin(\phi) \, d\phi
\]

\[
F_{y} = \frac{\gamma_{L_{0}}}{2\mu_{0}} \int_{0}^{\pi} \left( B_{s}^{2} - B_{q}^{2} \right) \sin(\phi) + 2B_{s}B_{q} \cos(\phi) \, d\phi
\]

where $B_{r}$ and $B_{\phi}$ are the resultant radial and circumferential components of the air gap flux density due to the PMs and EMC as expressed in Equation (34), $B_{r PM}$ and $B_{\phi PM}$ are defined in Equation (11) and $B_{r E}$ and $B_{\phi E}$ are given in Equation (31). It worth mentioning again that effect of the armature current is considered in the boundary condition required for calculating the distribution of EMC.

\[
B_{r} = B_{r PM} + B_{r E}
\]

\[
B_{\phi} = B_{\phi PM} + B_{\phi E}
\]

### 2.8 Modelling of dynamic eccentricity

It is easy to extend the developed model to consider dynamic eccentricity. In the dynamic eccentricity condition, the rotor rotates around the stator centre, $O_{s}$ (see Figure 1a). Therefore, to extend the model, it is enough to follow the following steps:

- Shifting the armature current by the angle of $-\theta_{r}$
- Finding the flux density components using the same procedure discussed above for the static eccentricity
- Shifting the computed flux density components by the angle of $\theta_{r}$

### 3 MODEL VERIFICATION

To evaluate the accuracy of the developed model, the field distribution due to PMs and armature current are obtained and compared with the results of finite element analysis (FEA). The design parameters of the considered consequent pole machine are given in Table 1. It should be noted that the magnetic axis of phase A and the direction of the static eccentricity are along the x-axis.

The flux density components due to PMs and the armature currents ($i_{a} = 1A$, $i_{b} = i_{c} = -0.5A$) at the zero rotor position are obtained by the analytical model and FEA given in Figure 3.

In the next tests, the UMF components at no-load and on-load conditions are obtained. For the on-load condition, a three-phase set of the balanced currents with the amplitude of 5A and the initial phase of zero in phase A is considered in the armature windings, and the rotor is rotated at the synchronous speed. The no-load and on-load UMF components are obtained by means of FEA and analytical model and given in Figure 4. From Figure 4a, it is observed that the armature reaction has a strong effect on the UMF (changing $F_{x}$ from 100 to 200N). In addition, the electromagnetic torque in the on-load condition is computed and compared by FEA in Figure 5. As seen in Figures 3–5, there is a good consistency between the results of the analytical model and FEA. Hereafter, the provided model is used to find the influence of the $d$-$q$ components of the armature current on the electromagnetic torque and UMF of the CPPM machines.


<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pole pairs</td>
<td>$p$</td>
<td>4</td>
</tr>
<tr>
<td>Number of slots</td>
<td>$Q_s$</td>
<td>36</td>
</tr>
<tr>
<td>Rotor radius</td>
<td>$R_r$</td>
<td>27.80 mm</td>
</tr>
<tr>
<td>Magnet radius</td>
<td>$R_m$</td>
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</tr>
<tr>
<td>Air gap length</td>
<td>$g$</td>
<td>0.70 mm</td>
</tr>
<tr>
<td>Inner stator radius</td>
<td>$R_s$</td>
<td>34.50 mm</td>
</tr>
<tr>
<td>Rotor slot arc to pole pitch</td>
<td>$a_r$</td>
<td>0.555</td>
</tr>
<tr>
<td>PM arc to pole pitch</td>
<td>$a_{PM}$</td>
<td>0.475</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>$D$</td>
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</tr>
<tr>
<td>Stack length</td>
<td>$L_{stk}$</td>
<td>100 mm</td>
</tr>
</tbody>
</table>

### 4 | EFFECT OF THE ARMATURE REACTION

#### 4.1 | Mathematical investigation

Since Equations (32) and (33) do not include explicitly the $dq$ currents, it is worth to investigate the physics of these variables by using other presentations to reveal the influence of the $dq$ components of the armature currents. The electromagnetic force due to the interaction of the flux density of rotor and the armature current could be obtained by the Lorenz force as given in Equation (35).

$$ F = F_r + F_\phi = L_{stk} i a_z \times B_r $$ (35)

where $F_r$ and $F_\phi$ are the radial and circumferential components of the Lorenz force, respectively, $B_r$ is the flux density due to the PMs, and $i$ is the armature current and $a_z$ is the unitary vector which is along with the z-axis (inside the paper). It is well-known that ($F_r$ and $F_\phi$) are responsible for UMF and electromagnetic torque, respectively. In Figure 6, the distribution of the rotor flux density and the relative position of the stator $dq$ currents are shown. As presented in Figure 6, the PM flux line crossed the $d$- and $q$-components of the armature current in the circumferential and radial directions, respectively. Therefore, based on Equation (35) and using the right-hand law, the interaction of the rotor flux density with the $d$- and $q$-components of the armature current results in UMF and electromagnetic torque, respectively, as given in Equation (36).

$$ UMF \propto L_{stk} i_d a_z \times B_r $$

$$ T_e = L_{stk} i_q a_z \times B_r $$ (36)

#### 4.2 | Simulation

To verify the discussed impact of the $d$- and $q$-components of the armature current on the electromagnetic torque and UMF in the CPPM machines, some simulations are carried out with different values of armature load angles and considering the machine data given in Table 1. It should be noted that against the SPM and PM-inset machines, in CPPM machines the distribution of the rotor flux density due to the PMs does not have odd half-wave symmetry, because the arc span of the PM and the rotor tooth could be different. Therefore, the $d$-axis position is not the...
In the magnetic hines is phase considered that along is a, in hine a load this phase is of point In flux hines middle the of linkage a, when linkage the no rotor 11.5 mac of the magnet. In the phase this is the axis rotor along ords position along – axis – a of point maximized, load of point when of rotor mac of the rotor the zero po-

The UMF at no-load and on-load conditions. (a) x, and (b) y-component.

The electromagnetic torque at on-load condition.

The influence of the d-axis current ($I_d = 0$) on the (a) electromagnetic torque and (b) x-component of the UMF.

position of the middle point of the rotor magnet. In the CPPM machines, the $d$-axis is along with the rotor point that when this point is along the magnetic axis of phase $a$, the no-load flux linkage of the phase $a$ is maximized. In other words, in the CPPM machines, the rotor $q$-axis is along with the rotor point that when this point is along the magnetic axis of phase $a$, the no-load flux linkage of the phase $a$ is zero. In the considered CPPM machine the position of the rotor $d$-axis is 11.5° away from the middle point of the PMs.

In the first test, the armature $q$-axis current is fixed at zero, and the waveforms of the electromagnetic torque and $x$-component of UMF ($F_x$) are obtained for different values of $d$-axis current as shown in Figure 7a,b, respectively. Since, the considered eccentricity is along the $x$-direction, $F_x$ is the dominant component of the UMF and for saving the space only $F_x$ is shown in Figure 7b.

In the next simulation, the armature $d$-axis current is fixed at zero and the waveforms of the electromagnetic torque and UMF $x$-component are obtained for different values of the $q$-axis current as shown in Figure 8a,b, respectively.

From the obtained results, it is found that, in the CPPM machine, the same as surface mounted PM-machines, the electromagnetic torque depends significantly on the $q$-axis current while it does not change very much with the changes in the $d$-axis armature current. In addition, it is found that the generated UMF is almost independent of the $q$-axis current while it is strongly affected by the $d$-axis armature current.
4.3 Flux weakening approach

As shown in Figure 7(b), the UMF of the eccentric CPPM machine could be reduced by using negative values of the d-axis armature current (flux weakening approach). Alleviation of UMF has the beneficiary of producing lower acoustic noise and smaller mechanical stress on the bearings of the eccentric CPPM machine. Although, injecting negative values of \( I_d \) results in a lower power factor, this strategy may be used for a short period of working between the detection of the eccentricity fault and the next closest possible planned maintenance.

To show the capability of reducing UMF by applying flux weakening strategy in the CPPM machine, two working conditions with similar electromagnetic torque is simulated by the provided model. In the first condition, the zero d-axis strategy is considered. In such a condition 6A current for the q-axis is required to generate 2 Nm average electromagnetic torque. In the other case, the flux weakening strategy with \( I_d = -4A \) is considered and to provide \( T_e = 2 \) Nm, 5A as the q-axis current is required. The electromagnetic torque and the UMF components for the given armature \( d-q \) currents are shown in Figure 9, for different values of the off-centre distance, \( D \). It is seen that applying the flux weakening strategy reduces the UMF of the eccentric CPPM machine without affecting the electromagnetic torque. In addition, from Figure 8 it is seen that the off-centre distance, \( D \), has no significant influence on the electromagnetic torque, while it affects strongly the machine UMF.

![Figure 8](image1.png)

**Figure 8** The influence of the q-axis current \((I_q = 0)\) on the (a) electromagnetic torque and (b) x-component of the UMF

![Figure 9](image2.png)

**Figure 9** Behaviour of the CPPM machine with static eccentricity for \( I_d = 0 \) and flux weakening approaches (a) \( T_e \) (b) \( F_x \) and (c) \( F_y \)

The same simulations, that is, considering two cases with \( (I_d,I_q) = (0,6)A \) and \( (I_d,I_q) = (-4,5)A \), are carried out by considering the dynamic eccentricity condition with \( D = 0.32 \) mm. The obtained torque, \( x \) and \( y \) components of the UMF and its magnitude are illustrated in Figure 10. Comparing the torque curves in Figures 9 and 10, it is found that the type of eccentricity does not change the average value of the electromagnetic torque, while the waveforms of \( x \) and \( y \) components of the UMF are affected significantly by the type of eccentricity, as expected. In addition, the same as the static eccentricity condition, the flux weakening approach will reduce the magnitude of the produced UMF.

5 CONCLUSION

An analytical 2D exact model is proposed for CPPM machines. The proposed model is capable of considering rotor eccentricity. In the modelling process, the air gap flux density components at no-load and on-load conditions are obtained by using a bilinear transformation, EMC and the subdomain
analysis methods. The validity of the proposed model is verified by means of FEA. In addition, using the provided model, the dependency of the electromagnetic torque and UMF on the armature d- and q-axis currents is investigated. It was found that the electromagnetic torque and the generated UMF are strongly affected by the q- and d-axis currents, respectively. Using the provided model, it is shown that the UMF of the eccentric CPPM could be diminished by applying a flux weakening approach.

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