Single Channel Multiple Signal Classification Using Pseudo-Doppler

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***Abstract*—An algorithm is presented to determine the number and direction of multiple indistinguishable radio frequency emitters using a single channel circular antenna array. This algorithm combines pseudo-Doppler direction finding with the MUSIC algorithm. The theory is provided along with a simulation. The simulation shows that the algorithm performs very well, with accuracy of better than one degree for typical scenarios, even with low signal to noise ratios.**

***Index Terms*—Array signal processing, direction-of-arrival estimation, Doppler measurement, multiple signal classification, signal detection.**

I. Introduction

Radio frequency (RF) direction finding (DF) has been an active area of research and development since the second world war. With telecommunications considered as critical infrastructure in many countries, it is important to rapidly find sources of RF interference. However, DF device portability can be a limitation when localising sources of interference, so low size, weight, and power (SWaP) devices are desirable.

In low SWaP contexts, circular arrays offer multiple advantages over linear arrays. They are generally of smaller size than equivalent linear counterparts and incident signals may be detected with azimuth angles across 360 degrees rather than the 180 ° coverage capability of linear arrays [1]. Furthermore, whilst this study's focus is 2D (2-dimensional), circular arrays are more adaptable to 3D (3-dimensional) problems as they can concurrently measure the elevation angle of an emitter that is not on the ground [2], [3]. They are typically less sensitive to coupling effects relative to linear designs [4] and have higher measurement accuracy due to the omni-directional symmetry of the array [5].

Despite the benefits of circular arrays [6], [7], algorithmic complexity and associated computational demands remains a fundamental challenge in DF, especially for portable devices. It is desirable to reduce the size and power requirements associated with DF systems, but this frequently requires compromise in their design due to tradeoffs with accuracy, expense, and efficiency. These considerations are even more relevant when considering problems which extend beyond the simplest use-cases, for example, DF for multiple emitters rather than a single emitter, or 3D rather than 2D determination of signals' angle of arrival (AOA). Diversity in available algorithms allows for optimisation of the design against application priorities. In particular, fast algorithms with low processing and memory overheads may be especially useful in any power-restricted environment, such as miniaturised wearable devices or on-board processing for UAVs or satellites.

Single channel receivers are a favourable option to reduce power consumption as they reduce the size, weight, complexity and cost of a system from the hardware perspective, and provide opportunities for efficiency in computational methods [8], [9]. Such receivers employ switching to sequentially sample antennas from an array, with classical single channel algorithms for direction finding including Watson-Watt and pseudo-Doppler [10]. Watson-Watt uses an amplitude comparison across orthogonally-oriented directional antennas to find AOA, whereas the pseudo-Doppler method is phase-based; in general, phase-based methods are more accurate as they are less susceptible to errors arising from factors such as noise. Pseudo-Doppler leverages the Doppler shift which is detectable from rapidly sampling around a circular array in a sequential manner to determine AOA for a single emitter [10]. Various generalisations of the methodology in [10] have since been developed to incorporate Doppler effects from switching for both linear and circular arrays [1], [11], [12]. A central theme of the extensions to [10] has been to improve upon the 5% uncertainty associated with the original pseudo-Doppler algorithm [13], including increasing its accuracy in multipath environments [14], [15]. For example, a Doppler-derived phase-lock loop AOA algorithm claiming modest performance improvements over [10] was provided in [16], and an extension to pseudo-Doppler using eigenspace methods and a phase center correction to account for antenna coupling was developed by [1]. Experimental studies explore diverse applications ranging from vehicle localisation [14] to wildlife tracking [17], [18]. However, the majority of single channel Doppler DF literature, whether theoretical or experimental, emphasises the study of single emitter methods and settings [19], [20], [21] with only limited exploration of the multiple emitter case, see for example [1], [9], which incorporates the results of [10] into a multi-emitter AOA algorithm.

When considering the multiple emitter problem, AOA algorithms include highly accurate but also computationally demanding maximum likelihood methods [22], through to techniques derived from algebraic subspaces such as MUSIC (noise subspace) [23] and ESPRIT (signal subspace) [24]. In

Manuscript received 7 August 2023; revised 13 October 2023; accepted 16 October 2023. Date of publication 26 October 2023; date of current version 9 November 2023. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Jean-Pierre Delmas. *(Corresponding author: Samuel P. Drake.)*

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Digital Object Identifier 10.1109/LSP.2023.3327899

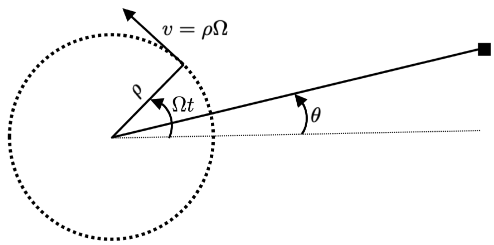

Fig. 1. Single receiver moving in uniform circular motion. The radius of the circle is ρ and the receiver's speed is $v = \rho\Omega$. The path travelled by the receiver is shown by the dashed line. A monotone emitter (shown by ■) is in the "far-field" in the direction θ.

applications where processing expenses associated with probabilistic methods are prohibitive, subspace methods are considerably more efficient for a minor accuracy tradeoff. Since its introduction for linear arrays in the 1980s, MUSIC has been adapted to further reduce computational costs [25], [26] and improve its applicability for different array layouts, including circular [2], [3]. However, in the context of a pseudo-Doppler approach MUSIC has rarely been implemented outside of single emitter scenarios, with the exception of a two emitter experiment within [1], potentially due to challenges associated with geometric ambiguities [27].

Here, we present an algorithm for determining the direction of multiple RF emitters with the same carrier frequency based only on phase measurements from a single channel circular array. A number of realistic assumptions are made to simplify the derivation. Consistent with other implementations of MUSIC [28], [10], etc, the assumptions are:

1) The emitters and receive antennas lie in a two-dimensional space.
2) The emitters are stationary.
3) The signals travel at a constant speed c.
4) There is no coupling between antennas.
5) The spacing between antennas is sufficiently small compared to the spacing between the antennas and any emitter so that the received signal amplitude from any one emitter is the same at all antennas.
6) The phase at each emitter is random for each "snapshot".
7) The total number of emitters K is less than the number of antenna elements M, i.e., $K < M$.

II. Single Emitter Direction Finding

This section reviews how to find the direction of one emitter from a receiver rotating in uniform circular motion, and then from a single channel uniformly spaced circular antenna array.

A. Direction Finding From Frequency Measurements

Consider a radio frequency receiver moving in a circle of radius ρ at a constant speed v. For uniform circular motion, assumed for convenience to be in a counterclockwise direction,

$$v = \rho\Omega, \tag{1}$$

where Ω is the angular speed in radians per second (Fig. 1). Assuming that a stationary emitter located well outside the circle emits a single tone at frequency $f_c = \omega_c/2\pi$ then, due to the Doppler effect, the signal's frequency at the receiver is[1]

$$\omega_r(t) = \omega_c \cdot \left[1 - \frac{\rho\Omega}{c}\sin\left(\Omega t - \theta\right)\right]. \tag{2}$$

If the frequency of the received signal can be measured then the direction of the emitter can be determined by rearranging (2), viz.

$$\theta = \Omega t - \arcsin\left(\frac{c}{\rho\Omega}\left[1 - \frac{\omega_r(t)}{\omega_c}\right]\right). \tag{3}$$

Indeed, alignment of the antenna, emitter and circle center corresponds to zero Doppler shift.

B. Direction Finding From Discrete Phase Measurements

Consider the scenario of a rotating receiver described in Section II-A, except that now the phase is measured instead of the frequency. The recorded phase is the integral of the angular frequency, so from (2)

$$\begin{aligned}\phi(t) &= \int \omega_r(t)dt \\ &= \omega_c\left[t + \frac{\rho}{c}\cos(\Omega t - \theta)\right] - \omega_c\frac{\rho}{c}\cos\theta + \phi^0 \\ &= \omega_c t + \frac{\omega_c\rho}{c}\left[\cos(\Omega t - \theta) - \cos\theta\right] + \phi^0 \\ &= \omega_c t + \varphi(t) + \phi^0,\end{aligned}$$

where the constant of integration is chosen so $\phi(0) = \phi^0$ and

$$\varphi(t) = \frac{\omega_c\rho}{c}\left[\cos(\Omega t - \theta) - \cos\theta\right] \tag{4}$$

is referred to as the *baseband* phase.

When the signal is measured at M discrete uniformly spaced time instants then the baseband phase is

$$\varphi^m \equiv \varphi(mt) \tag{5}$$

$$= \frac{\omega_c\rho}{c}\left[\cos(\Omega m\Delta t - \theta) - \cos\theta\right], \tag{6}$$

where $m = \{0, 1, \ldots, M-1\}$ and Δt is the time between samples so that the mth measurement occurs at $t = m\Delta t$.

If the carrier angular frequency ω_c, receiver angular speed Ω, and radius of the circle ρ are known, then emitter direction θ and nuisance parameter ϕ^0 can be determined by fitting the measured baseband phase data to the right hand side of (6).

C. Pseudo-Doppler Direction Finding

Consider the successively sampled phase measurements of a stationary circular antenna array, shown in Fig. 2, instead of the discretely sampled mechanically rotating receiver. We show hereafter that the successive activation of each antenna every Δt seconds is equivalent to the rotating receiver in Section II-A and II-B when $M = 2\pi/(\Omega\Delta t)$. Assuming that when $m = 0$ the phase at the emitter is ϕ^0 then the measured phase at the m-th antenna, measured at time $m\Delta t$, is

$$\phi^m = \omega_c\left[m\Delta t - l_m/c\right] + \omega_c l_0/c + \phi^0, \tag{7}$$

[1]This assumes that $\rho\Omega/c \ll 1$. If this is not the case special relativistic corrections to the Doppler shift formula are required. In virtually all situations of practical interest, the condition obviously is met.

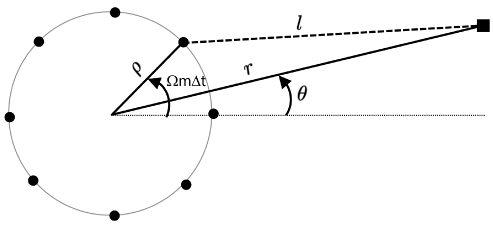

Fig. 2. Diagram of circular pseudo-Doppler antenna array of radius ρ. The locations of each antenna are shown by • and the emitter by ■. The angle and distance to the emitter are θ and l respectively.

where l_m is the distance between where the m-th measurement is made and the emitter.

From Fig. 2 and the *Law of Cosines*, distance l_m is

$$\begin{aligned} l_m &= \sqrt{r^2+\rho^2-2\rho r\cos\left(\Omega m\Delta t-\theta\right)} \\ &= r\sqrt{1+\rho^2/r^2-2\rho/r\cos\left(\Omega m\Delta t-\theta\right)} \\ &\approx r\left[1-\rho/r\cos\left(\Omega m\Delta t-\theta\right)\right] \\ &= r-\rho\cos\left(\Omega m\Delta t-\theta\right) && (8) \\ &= l_0-\rho\cos\left(\Omega m\Delta t-\theta\right)+\rho\cos\theta, && (9) \end{aligned}$$

where ρ is the radius of the circular antenna array, r is the distance from the center of the array to the emitter, and we have made use of the far field assumption that $\rho/r \ll 1$.

The phase at the m-th antenna is therefore

$$\begin{aligned} \phi^m &= \omega_c\left[m\Delta t-l_m/c\right]+\omega_c l_0/c+\phi^0 \\ &\approx \omega_c m\Delta t+\frac{\omega_c\rho}{c}\left[\cos\left(\Omega m\Delta t-\theta\right)-\cos\theta\right]+\phi^0 \\ &= \omega_c m\Delta t+\varphi^m+\phi^0, \end{aligned}$$

where

$$\varphi^m = \frac{\omega_c\rho}{c}\left[\cos\left(\Omega m\Delta t-\theta\right)-\cos\theta\right]. \qquad (10)$$

Notice that the baseband measured phase in (10) is identical to the baseband phase in (6)! Consequently the method for obtaining the direction of the emitter is identical to that described in Section II-B. The parity between these methods for finding the direction of the emitter is why the method described in this section is known as the *pseudo-Doppler* technique for direction finding.

III. Pseudo-Doppler Multiple Emitter Direction Finding

In this section we present an algorithm for determining the directions of multiple indistinguishable emitters, assumed to be coplanar with the antenna array by combining MUSIC with Pseudo-Doppler direction finding. The emitters are indistinguishable in that they have the same carrier frequency. However, it is assumed that they lie in different directions, i.e. no two or more are collinear with the array center. The emitters may have the same amplitudes and modulation schemes, but the content of the messages differ. We consider N "snapshots" consisting of M measurements around the antenna array as described in Section II-C. In order for each snapshot to begin with independent (i.e. uncorrelated) phase measurements from the emitters, it is necessary for the time gap between each snapshot to be sufficiently long. This is also a requirement for the standard linear array MUSIC algorithm.

Consider the m-th antenna in the circular array described in Section II-C and receiving signals from multiple emitters at a particular instant in time $m\Delta t$. The complex representation of the baseband of the received signal is

$$x^m = \left[\sum_{k=1}^{K}\alpha_k e^{j\phi_k^0}e^{j\varphi_k^m}\right]+n^m, \qquad (11)$$

where n^m is zero mean unit variance complex Gaussian white noise, K is the number of emitters, α_k is the amplitude of the signal from the k-th emitter,[2] ϕ_k^0 is the phase measured at the first antenna of the k-th emitter, and

$$\varphi_k^m = \frac{\omega_c\rho}{c}\left[\cos\left(\Omega m\Delta t-\theta_k\right)-\cos\theta_k\right], \qquad (12)$$

where θ_k is the direction of the k-th emitter.

With M antennas, the linear algebra from of (11) is

$$\boldsymbol{x} = \boldsymbol{A}\boldsymbol{s}+\boldsymbol{n}, \qquad (13a)$$

where

$$\boldsymbol{x} = \begin{bmatrix} x^0 \\ x^1 \\ \vdots \\ x^{M-1} \end{bmatrix}, \quad \boldsymbol{A} = \begin{bmatrix} 1 & \cdots & 1 \\ e^{j\varphi_1^1} & \cdots & e^{j\varphi_K^1} \\ \vdots & \ddots & \vdots \\ e^{j\varphi_1^{M-1}} & \cdots & e^{j\varphi_K^{M-1}} \end{bmatrix} \qquad (13b)$$

$$\boldsymbol{s} = \begin{bmatrix} \alpha_1 e^{j\phi_1^0} \\ \alpha_2 e^{j\phi_2^0} \\ \vdots \\ \alpha_{M-1} e^{j\phi_K^0} \end{bmatrix} \quad \text{and} \quad \boldsymbol{n} = \begin{bmatrix} n^0 \\ n^1 \\ \vdots \\ n^{M-1} \end{bmatrix}. \qquad (13c)$$

Equation (13) is the standard MUSIC equation as described in Speilman et. al [28]. In our case the "action vectors" (which are column vectors of $\boldsymbol{A}$) are

$$\boldsymbol{a}(\theta) = \begin{bmatrix} 1 \\ e^{j\frac{\omega_c\rho}{c}\left[\cos(\Omega\Delta t-\theta)-\cos(\theta)\right]} \\ \vdots \\ e^{j\frac{\omega_c\rho}{c}\left[\cos(\Omega[M-1]\Delta t-\theta)-\cos(\theta)\right]} \end{bmatrix}, \qquad (14)$$

so that

$$\boldsymbol{A} = \left[\boldsymbol{a}(\theta_1),\ldots,\boldsymbol{a}(\theta_M)\right]. \qquad (15)$$

The procedure for estimating the emitters' directions is:

1) Concatenate $N(>M)$ received signals vectors to create the $M\times N$ matrix

$$\boldsymbol{X} = \left[\boldsymbol{x}_1\ldots\boldsymbol{x}_N\right].$$

2) Use $\boldsymbol{X}$ to generate the $M\times M$ Hermitian matrix

$$\hat{\boldsymbol{R}} = \frac{1}{N}\sum_{i=1}^{N}\boldsymbol{x}_i\boldsymbol{x}_i^* = \frac{1}{N}\boldsymbol{X}\boldsymbol{X}^*.$$

[2] We are assuming that the amplitude at each antenna is the same. This assumption is equivalent to assuming that the emitter is far away (relative to the spacing of the antennas) and that there is no coupling between the antennas.

Note here that the superscript asterisk * denotes the complex conjugate transposed.

3) Calculate the M real eigenvalues and eigenvectors of $\hat{\boldsymbol{R}}$:

$$\hat{\boldsymbol{R}} = \sum_{i=1}^{M} \lambda_i \hat{\boldsymbol{v}}_i \hat{\boldsymbol{v}}_i^*$$

where $\lambda_1 \geq \lambda_2 \ldots \geq \lambda_M \geq 0$ and $\{\hat{\boldsymbol{v}}_1 \hat{\boldsymbol{v}}_2, \ldots, \hat{\boldsymbol{v}}_M\}$ are the ordered sets of eigenvalues and their eigenvectors.

4) Determine the number of emitters and the eigenvectors associated with the "signal subspace": given a threshold value λ_T, the number of emitters is the number of values of λ_i greater than λ_T. If there are K emitters then the signal subspace's eigenvectors are $\hat{\boldsymbol{v}}_1, \ldots, \hat{\boldsymbol{v}}_K$.[3]

5) Determine the eigenvectors associated with the "noise subspace"; these eigenvectors are $\hat{\boldsymbol{v}}_{K+1}, \ldots, \hat{\boldsymbol{v}}_M$.

6) Calculate the noise subspace cost function for all possible emitter directions[4], i.e. $\theta = [0, 2\pi)$ for cost function

$$\Phi(\theta) = \frac{\boldsymbol{a}^*(\theta)\boldsymbol{a}(\theta)}{\sum_{j=K+1}^{M} \left[\boldsymbol{a}^*(\theta)\hat{\boldsymbol{v}}_j\right]^2}. \quad (16)$$

7) The directions of the K emitters peaks are found with a standard peak-finding routine on the cost function $\Phi(\theta)$.

IV. Simulation

The algorithm described in Section III was implemented using the following parameters:

- There were four emitters, $K = 4$.
- Each emitter has a constant carrier frequency of 300 MHz, $\omega_c = 600\pi$ radians per second.
- Each emitter had the same amplitude, i.e. $\alpha_k = 1 \forall k$.
- For each snapshot each emitter had a new independent random phase uniformly distributed from 0 to 2π, i.e. $\phi_k^0 \sim 2\pi U(0, 1)$.
- The circular antenna array had 12 receivers, $M = 12$.
- The radius of the circular antenna array was 1 m, $\rho = 1$.
- The array sampling rate was 12 KHz, $\Delta t = \frac{10^{-3}}{12}$.
- The noise on each receiver was $\sigma = 1$; for four emitters this equated to a signal-to-noise ratio (SNR) of -6 dB.
- There were 20 "snapshots", $N = 20$.
- The threshold eigenvalue is set to the 95 percentile, $\lambda_T = (1.96\sigma)^2 = 3.84$.

The cost function $\Phi(\theta)$ for this simulation is shown in Fig. 3, which shows that cost function peaks agree with the true emitter locations. In this simulation the rounded eigenvalues of $\hat{\boldsymbol{R}}$ are, in descending order, $\lambda_i = \{9, 13, 10, 6, 2, 1, 1, 1, 1, 0, 0, 0\}$. The first four eigenvalues are above the threshold of $(1.96\sigma)^2$, so the number of emitters is correctly estimated as four. The true emitter locations were $\theta = \{45.71, 93.77, 323.01, 348.02\}$, whereas the peaks in the cost function were at $\hat{\theta} = \{45.13, 94.26, 322.90, 347.97\}$. The estimated directions are within 1° of the true directions, despite the low signal-to-noise ratio; bootstrapping the simulation 1000 times gives a root-mean-square-error (RMSE) of $2.3^\circ \pm 0.71^\circ$ for the predicted emitter locations. Qualitatively, we note that the ambiguities arise when two or more emitters are close together within approximately 2° of each other. Increasing the system noise raises the threshold eigenvalue, which may reduce the number of predicted emitters from four to three or two (Table I), but has negligible impact on the RMSE, likely due to the fact that the ambiguities arise from inherent symmetries in the array [27]. All four of the emitters were detected over 90% of the time for SNR values better than -5.5 dB, and 73% of the time for the worst case SNR of -6 dB (Table I).

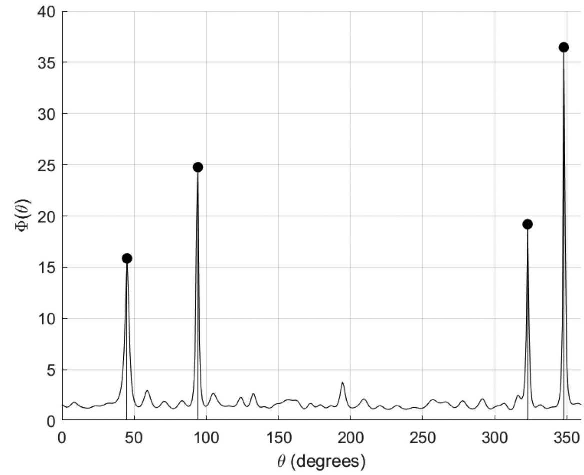

Fig. 3. Cost function $\Phi(\theta)$ given by (16) for the simulation parameters described in Section IV. The stem plot shows the true directions of the emitters. The peaks in the cost function are in excellent agreement with the true emitter directions.

TABLE I
Probability of Detecting n Emitters At Different Signal-to-Noise Ratios (SNR)

SNR (dB)	**-5**	**-5.25**	**-5.5**	**-5.75**	**-6**
# detections					
4	0.96	0.94	0.92	0.85	0.72
3	0.042	0.058	0.077	0.15	0.28
2	0	0	5.0×10^{-4}	5.0×10^{-4}	4.0×10^{-3}
≤1	0	0	0	0	0

Probability of detecting n emitters above the threshold eigenvalue at different signal-to-noise ratios. Each SNR was bootstrapped 2000 times and the probability vector was obtained from the predicted number of emitters in each iteration. The true number of emitters $K = 4$, and all other parameters are as described in section IV.

V. Conclusion

An algorithm has been presented to determine the direction of multiple indistinguishable radio frequency transmitters. The algorithm uses a single channel to sample multiple antennas in a circular array. Simulations show that this method is able to determine the directions of four emitters to an accuracy of less than one degree from 12 antenna elements given 20 snapshots. Future work will involve building the antenna array and testing it in a realistic environment.

[3]Alternative methods for estimating the number of emitters include Bayesian [29] and Akaike [30] Information Criteria.

[4]The numerator in equation (6) in [28] is $\boldsymbol{a}(\theta)\boldsymbol{a}^*(\theta)$, which appears to have the complex conjugate in the wrong place.

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